$O(R^{-1/10})$. This value of ϵ does not agree with the earlier result $\epsilon = O(R^{-2/5})$ proposed by Stewartson.[4] His derivation neglected displacement effects, and the external pressure distribution was prescribed to be the pressure distribution given by Taylor's solution for a weak shock wave. However, it can be shown that the pressure perturbation resulting from the outward streamline displacement in Stewartson's solution is of order $\epsilon^{1/6}$, much larger than the imposed pressure rise of order ϵ. This observation tends to support the present description in terms of a free interaction. The pressure rise of order $R^{-1/5}$ then presumably corresponds to the first appearance of separation. In transonic small-disturbance theory, the important similarity parameter can be interpreted as the ratio of $M_\infty^2 - 1$ to a typical pressure change. For the present case the corresponding similarity parameter may be written as $(M_\infty^2 - 1)/R^{-1/5}$.

As in the case of higher Mach numbers, increasing the shock strength would be expected to lengthen the region of separated flow and thus to increase the distance between the separation point and the main part of the pressure increase. Therefore one is led to the assumption that a shock strength of order $R^{-1/5}$ at transonic speeds also corresponds to the first appearance of shock bifurcation. For $R^{-1/5} \ll \epsilon \ll 1$ a pressure rise of order $R^{-1/5}$ accompanied by separation would be expected to occur at the base of the forward branch of a lambda shock, over a streamwise distance of order $R^{-3/10}$. Since the equations do not seem to permit a pressure rise of this order in any shorter region including the separation point, it appears that immediately outside the boundary layer the outgoing compression waves have not yet coalesced to form a shock. This conclusion also seems to be implied by the schlieren photographs and surface-pressure measurements of Refs. 6 and 7. The description of separation therefore could be carried out by numerical solution of the sublayer equations as in Ref. 1, with the transonic approximation relating p and θ to replace the linear-theory relation just outside the boundary layer. Since the pressure gradient associated with incoming waves is expected to be much smaller, the required condition is found by expanding the simple-wave solution for $M \rightarrow 1$; the result is $p \sim -\gamma(\gamma + 1)^{-1/3}(\frac{3}{2})^{2/3}(\theta + \nu_\infty)^{2/3} + \gamma(\gamma + 1)^{-1}(M_\infty^2 - 1)$, where $\nu_\infty = -(\frac{2}{3})(\gamma + 1)^{-1}(M_\infty^2 - 1)^{3/2}$. The distance downstream to the main branch of the lambda shock increases as ϵ increases, and the main branch is directly followed by an expansion.[6,7] The large rise in surface pressure therefore occurs slightly downstream of the main shock; if the boundary layer reattaches, the over-all flow pattern is similar in many respects to that observed at higher Mach numbers.

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Nonequilibrium Dissociating Nitrogen Flow over a Circular Cylinder

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1. Introduction

When the kinetic energy of a high-speed stream of diatomic gas is near the dissociation energy for the gas, a body placed in the stream can cause dissociation to occur in the stagnation region by virtue of the bow shock wave that thermalizes much of the kinetic energy. The dissociation rate can be characterized by the distance through which the gas has to move before reaching equilibrium. When the body size is similar to this characteristic dissociation distance, nonequilibrium effects have to be taken into account. Such flows are of interest in re-entry situations and have been studied extensively using various theoretical models.[1–4] Of particular interest, because of its practical relevance, is the flow of nitrogen over a blunt body. Until recently, nitrogen flows of sufficient speed and density could not be produced in the laboratory on a large enough scale to produce significant effects due to chemical reaction. Experimental studies of reacting hypersonic flows were restricted to gases with lower dissociation energies, such as oxygen (Spurk and Bartos[5]), or to free-flight ranges, in which flowfield measurements are difficult and expensive.

The recent completion of the large free-piston shock-tunnel at the Physics Department, A.N.U. made it possible to study reacting nitrogen flows in the laboratory, (Stalker and Hornung[6]). The first results of measurements in this tunnel, which were made using as a model a circular cylinder in cross flow, are described here, and compared with theoretical calculations. While this facility is capable of more than twice the flow speed of the present experiment, the conditions chosen give a reacting flow of good quality, with low freestream dissociation.

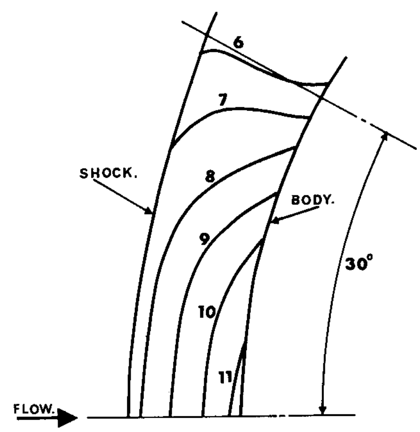

Fig. 1 Contours of density in reacting nitrogen flow over a 6-in.-diam circular cylinder. Calculation based on Freeman's model, at conditions of experiment. The numbers on the contours represent the density as multiples of freestream density.

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2. Experiment

The conditions in the reservoir and freestream of the test flow used for the present experiment were as follows:

1) Reservoir (thermodynamic equilibrium): specific enthalpy—1.56×10^{11} cm^2/sec^2; pressure—2.4×10^8 dyne/cm^2.

2) Freestream: speed = 5.2 km/sec; density = 2.5×10^{-6} g/cm^3; pressure = 8.3×10^3 dyne/cm^2; Mach number = 7.3; dissociation 3% N, 97% N_2 by mass; stream diameter —30 cm. The stream is steady for approximately 0.5 msec.

The flowfield over the model, a 6-in.-diam brass tube, 10-in. long, was recorded using a Mach Zehnder interferometer with a field of view of 8-in. diam. The interferometer mirrors were set for infinite fringe width before the flow was started, so that the fringes represent contours of optical path length. The gas in the stagnation region of the cylinder radiates quite strongly, so that a source very much brighter than the gas luminosity was needed. An exploding wire, 0.3-mm diam, vaporized by discharging through it a 500 μF capacitor charged to 3.5 kv proved to be a very good source for this purpose. It gave a bright flash of about 100 μsec duration. A band-pass filter (5330 ± 100 Å) was introduced just before the camera lens, so that steps from any fringe to the next correspond to equal differences of optical path.

3. Results

In order to see the influence of chemical reaction on the flowfield, a calculation was made, at the freestream conditions of the preceding experiment, using Freeman's model for reacting flow over a sphere, with modifications to allow for the more complicated form of the pressure field on a circular cylinder, and also for a more general form of the free stream conditions. The result is shown in Fig. 1 in the form of a plot of density contours. The reaction rate in this calculation was that given by Vincenti and Kruger[7] (see p. 231). This has to be compared with the two extreme cases, of flow with chemical equilibrium at one end, and flow with constant composition (frozen flow) at the other end of the range of reacting flows. Numerical methods for calculating blunt body flowfields are more straightforward in the axially symmetric case and, because this is also the case of practical interest, they have been developed more fully than for plane symmetry. One such method (Lomax and Inouye[8]) was used to calculate frozen and equilibrium flow for a 6-in.-diam sphere with the freestream conditions of the preceding experiment. The reacting flow was also calculated using Freeman's model as before. The result is shown in the form of contour plots of density in Fig. 2. The effect of having a finite reaction rate can be seen quite clearly. The frozen and equilibrium flows have density fields which are qualitatively quite similar to each other, the density being nearly constant in the stagnation region. The essential difference between them is that the density is considerably higher, and as a consequence the distance between the body and shock wave is smaller, in the equilibrium case. The reacting flow, however, is seen to have a density field that is quite different. The density rise from the shock to the body along the stagnation streamline is almost as high as the rise across the shock wave. The shape of the density contours is also quite different in the region around 25° from the stagnation point. Although the geometry of the experiment is not axially symmetric, the flow over a sphere gives a qualitative picture of the changes which are likely to occur in the density field of a circular cylinder in the transition from frozen to equilibrium flow.

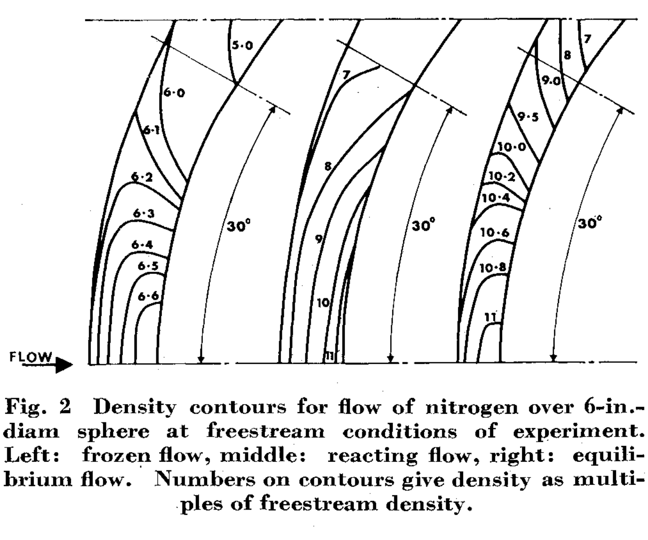

Fig. 2 Density contours for flow of nitrogen over 6-in.-diam sphere at freestream conditions of experiment. Left: frozen flow, middle: reacting flow, right: equilibrium flow. Numbers on contours give density as multiples of freestream density.

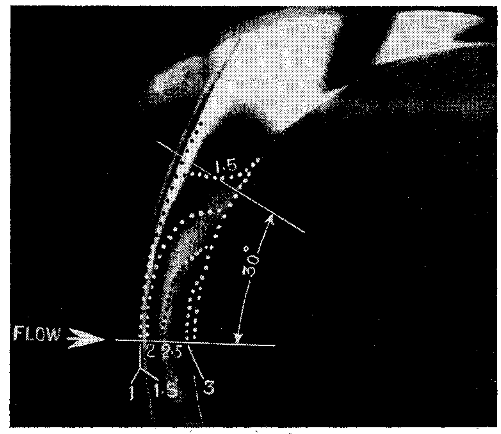

Fig. 3 Infinite fringe interferogram of reacting nitrogen flow over 6-in.-diam circular cylinder. Calculated fringe pattern is superimposed as dotted lines. The numbers on the dotted lines refer to fringe number relative to the freestream as datum. For freestream conditions, see text.

The interferogram obtained in the above experiment is shown in Fig. 3. In order to compare this with the theoretical result of Fig. 1, the fringe pattern was calculated from the theoretical distributions of density and atomic nitrogen concentration, using the refractivities of atomic and molecular nitrogen as given by Alpher and White.[9] The result is superimposed on Fig. 3. It is evident that the experimental and theoretical fringe patterns are not only very similar in form, but give quite close quantitative agreement in the stagnation region.

4. Conclusions

The large free piston shock tunnel at A.N.U. provides a good means of studying dissociating nitrogen flows in the laboratory. The case studied here, of flow over a 6-in.-diam cylinder, shows that the density field in the stagnation region is a sensitive indication of the extent to which the chemical reaction influences the flow. Freeman's model for reacting flow, used with accepted values for nitrogen dissociation and recombination rates, gives a satisfactory estimate of the density field as measured by optical interferometry.

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Stability of Heavy Circular Arches with Hinged Ends

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THE problem of buckling of nonshallow arches subjected to static loads can be subdivided into two classes: 1) arches whose undeformed centroidal line coincides with the funicular curve of the applied loading, and whose instability is preceded by small displacements; and 2) arches that buckle by sidesway or snap-through at large deflections. The first class has been investigated rather extensively (for a bibliography of fundamental problems see Refs. 1 and 2). Its problems are of the Euler type, amenable to comparatively simple analysis.

The problems of the second class are highly nonlinear and present great computational difficulties. Nevertheless, a considerable number of papers has appeared recently dealing with the stability of arches subjected to concentrated loads.[3–11] The problem of distributed loads on nonshallow arches with large deflections has not been investigated analytically, and it is the purpose of this Note to initiate research in that direction. Of practical interest are behaviors of arches with circular, parabolic, and catenarian centroidal curves subjected to dead weight, uniform and piecewise uniform vertical load, and combinations of various loads. However, the present investigation will be restricted to two-hinged circular arches of constant cross section buckling under their own weight.

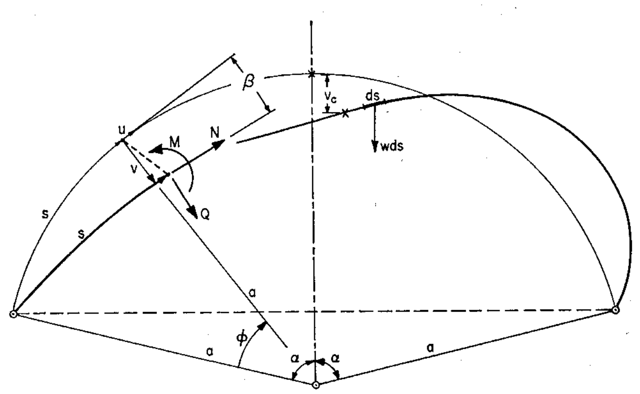

Fig. 1 Deformed and undeformed centroidal curve of the arch.

The theory of plane curved beams and arches with very large deflections has been presented in several publications, e.g., Refs. 8, 12, and 13. For that reason, only a summary of necessary equations is presented herein.

Assuming an inextensible centroidal curve—an assumption that is commonly made for nonshallow arches—we can write the differential equations of equilibrium in the form

$$M' = aQ \tag{1a}$$

$$aN' - (1 + \beta')M' = -a^2w \sin(\phi + \beta - \alpha) \tag{1b}$$

$$M'' + a(1 + \beta')N = -a^2w \cos(\phi + \beta - \alpha) \tag{1c}$$

where M is the bending couple; Q the shearing force; N the normal tensile force; w the dead weight of the arch rib, per unit length; a the constant radius of the undeformed centroidal curve; β the angle of rotation of a line element of the centroidal curve; ϕ the angle measured from a reference radius; and primes indicate differentiation with respect to ϕ (see Fig. 1). The bending couple M is related to β' by

$$M = -(EI/a)\beta' \tag{2}$$

according to the hypothesis of plane cross sections. Herein, E is the modulus of elasticity and I the moment of inertia of the cross-sectional area of arch rib.

The geometrical relations are given by

$$v' + u = a \sin\beta, \qquad u' - v = a(\cos\beta - 1) \tag{3}$$

where u and v are the tangential and normal displacement components of the centroidal curve (see Fig. 1).

The boundary conditions are

$$u = v = M = 0 \tag{4}$$

at the two hinges.

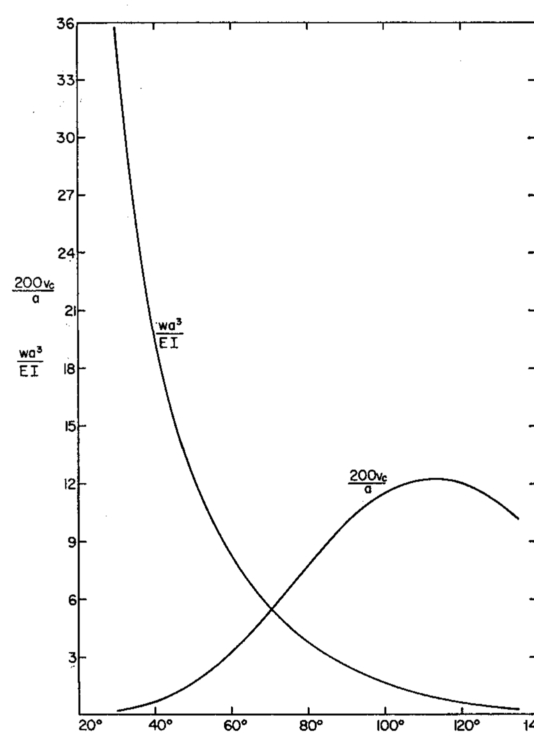

Fig. 2 Variation of the critical load w and critical vertical displacement v_c of the crown with the subtending angle 2α.

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